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STUDY OF REFLECTION COEFFICIENTS IN SELF –REINFORCED MEDIUM IN THE ABSENCE OF DISSIPATION

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ABSTRACT: When working with a poroelastic material that is self-reinforced, it is required to carry out calculations in order to arrive at an estimate of the reflection coefficients of the material. This can be accomplished by calculating the material's material properties. This technique is utilized in situations in which the substance does not undergo any dissipation. In the course of this procedure, the computation of the reflection coefficients of the shear SV wave, the rapid dilatational P-I wave, and the slow dilatational P-II wave is carried out at a number of different locations. This is done in order to ensure that precise results are obtained. This action is taken in order to guarantee that the procedure is carried out in the appropriate manner. The numerical results are obtained by utilizing MATLAB in order to achieve the purpose of acquiring them, which is the desired objective. This is the wanted objective. The graphs that are a part of the numerical session are utilized in order to exhibit these facts to the audience once they have been acquired at the conclusion of the numerical session.

Keywords: *Poroelastic material, Self-reinforced material, Reflection coefficients, Material properties calculation, Wave propagation, Shear SV wave*

1. INTRODUCTION

During the past half century in many scientific fields, the wave reflection and transmission phenomena have long history and received much attention such as Marine Seismology, Geotechnical Engineering, Acoustics, and Geophysics. Analysis of reflection and transmission phenomena can be used to understand the various materials. Sandstone is a great source for quartz which is very useful in day to day life. Sandstone deposits are may be cylindrical in shape and may be surrounded by poroelastic medium, soil or rock. The study of reflection and transmission of waves incidented at the interface gives the information pertaining to sandstone deposits. The reflection and transmission coefficients are generally based on the medium consisting of two non-homogeneous layers separated by a horizontal interface. In Acoustics, the said coefficients are used to understand the effect of various materials on their acoustic environments. In the research domains such as Geophysics, and Medicine, the pertinent analysis can be used as Non Destructive Evaluation (NDE) tool. Employing transfer matrix method, Bogy and Gracewski derived the plane wave reflection coefficient for a layered solid half-space, for an isotropic, homogeneous elastic layer. Sinha and Elsibai investigated the reflection of thermo elastic waves from the free surface of a solid half space and at the interface of two semi-infinite media in welded contact. For generalized thermo elastic medium, the reflection phenomena of SV-waves are discussed by Abd-Alla and Al-Dawy. In the paper, the reflection coefficient is calculated for shear wave that incident from within the solid on its boundary. Singh exploit the reflection of P and SV waves from the free surface of an elastic solid with generalized thermo-diffusion. Employing Biot's theory of poroelasticity, reflection of plane waves at boundaries of a poroelastic half space is discussed by Tajuddin and Hussaini. In the paper, it is clear that the overlapping parameter between solid and fluid plays a significant role in generating the reflected slow dilatational wave. Dai and Kaung calculated the reflection and transmission of elastic waves at the interface between water and double porosity solids. The analytical

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expressions of all the three phase velocities of qP, qSV and qSH waves has been derived by Chattopadhyay. Reuven Gordon explained the reflection of cylindrical surface waves.

In this the reflection coefficients of fast dilatational ($P-I$) wave, slow dilatational $(P-II)$ wave and the shear SV waves with angle of incidence are calculated.

2. FORMULATION AND SOLUTION OF THE PROBLEM

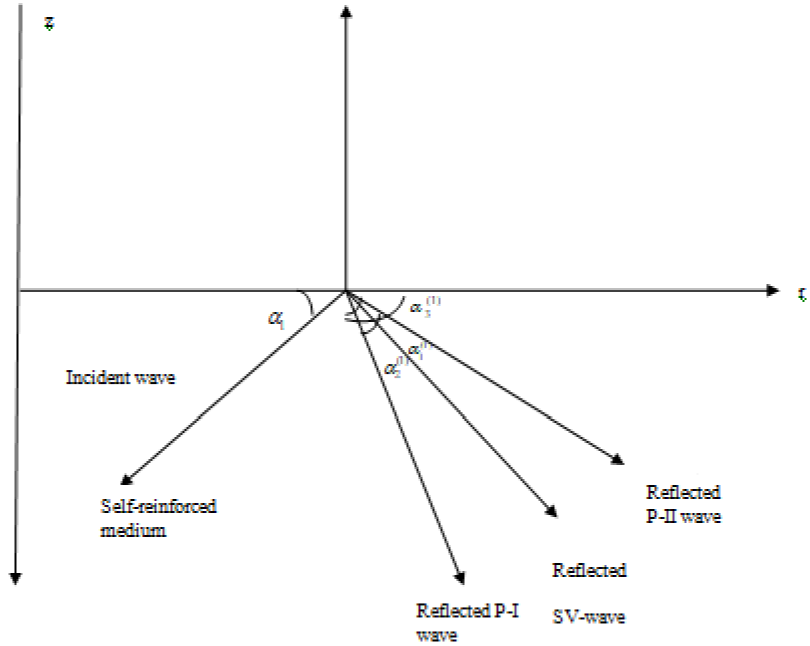

Fig. 1 Geometry of the problem

Consider a self-reinforced layer, If the wave is incidented in the reinforced layer two dilatational waves and a shear wave are either reflected or transmitted. In this problem reflection only calculated .The equations of motion of a homogeneous, isotropic poroelastic solid in absence of dissipation are

$$N\nabla^2\vec{u} + \nabla(Ae + Q\varepsilon) = \frac{\partial^2}{\partial t^2}(\rho_{11}\vec{u} + \rho_{12}\vec{U}),$$

$$\nabla(Qe + R\varepsilon) = \frac{\partial^2}{\partial t^2}(\rho_{12}\vec{u} + \rho_{22}\vec{U}).$$

where $\vec{u} = (u, v, w)$ and $\vec{U} = (U, V, W)$ are displacement vectors of solid and fluid, respectively, e and ε are the dilatations of solid and fluid, $P = A + 2N, Q,$ and R are all poroelastic constants, The mass coefficients ρ_{11}, ρ_{12} and ρ_{22} satisfy the relations

$$\rho_1 = \rho_{11} + \rho_{12} = (1-\beta)\rho_s,\ \rho_2 = \rho_{12} + \rho_{22} = \beta\rho_f,\ \rho = \rho_1 + \rho_2 = \rho_s + \beta(\rho_f - \rho_s).$$

Here ρ_1 and ρ_2 are mass of solid and fluid per unit volume of aggregate, ρ is total mass of solid-fluid aggregate per unit volume ρ_{11}, ρ_{12} and ρ_{22} are mass coefficients, ρ_s and ρ_f are mass density of the solid and fluid, β is the porosity of the aggregate. The solid stresses σ_{ij} and fluid pressure s are given by

$$\sigma_{ij} = 2Ne_{ij} + (Ae + Q\varepsilon)\delta_{ij}, \quad (i, j = 1,2,3), \qquad s = Qe + R\varepsilon. \ (2)$$

In eq. (2), δ_{ij} is the well-known Kronecker delta function.

For an axisymmetric waves, the displacement vectors of solid and fluid are $\vec{u}(u_1, 0, w_1)$ and $\bar{U}(U_1, 0, W_1)$, respectively. The stress tensor for a self-reinforced layer along preferred direction $\vec{a}$ is given [15]

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_l e_{kl}\delta_{ij} + a_i a_j e_{kk}) + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) + \beta a_k a_m e_{km} a_i a_j \quad (3)$$

here $i, j, k, l, m = 1,2,3$, σ_{ij} is the incremental stress component, e_{ij}'s are the strain components given by

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \text{ and } \vec{a} = (a_1, a_2, a_3)^T$$

is the preferred unit direction of reinforcement. The potential decomposition of displacements for pertinent upper self reinforced layer as follows:

$$u_1 = \frac{\partial \phi_1}{\partial r} - \frac{\partial \psi_1}{\partial z},\ U_1 = \frac{\partial \phi_2}{\partial r} - \frac{\partial \psi_2}{\partial z},$$
$$w_1 = \frac{\partial \phi_1}{\partial z} + \frac{\partial \psi_1}{\partial r} + \frac{\psi_1}{r},\ W_2 = \frac{\partial \phi_2}{\partial z} + \frac{\partial \psi_2}{\partial r} + \frac{\psi_2}{r}. \quad (4)$$

The displacement potentials ϕ's and ψ's which are functions of r, z, and t, and are assumed as.

$$\phi_1 = \begin{cases} \phi_{(k1)} + \phi_l^{(k)} = A_k e^{i(\omega t - \delta_{k1}(r\sin\alpha_k - z\cos\alpha_k))} + A_l^{(k)} e^{i(\omega t - \delta_{l1}(r\sin\alpha_l^{(k)} - z\cos\alpha_l^{(k)}))}, & l, k = 1,2, \\ \phi_{(3)} + \phi_l^{(k)} = A_k e^{i(\omega t - \delta_{k1}(r\cos\alpha_k - z\sin\alpha_k))} + A_l^{(k)} e^{i(\omega t - \delta_{t1}(r\sin\alpha_l^{(k)} - z\cos\alpha_l^{(k)}))}, & l, k = 3, \end{cases}$$

$$\phi_2 = \begin{cases} \phi_{(k)} + \phi_l^{(k)} = A_{k1} e^{i(\omega t - \delta_{k1}\mu_k(r\sin\alpha_k - z\cos\alpha_k))} + A_l^{(k)} e^{i(\omega t - \delta_{t1}\mu_{t1}(r\sin\alpha_l^{(k)} - z\cos\alpha_l^{(k)}))}, & l, k = 1,2, \\ \phi_{(3)} + \phi_{t1}^{(k)} = A_{k1} e^{i(\omega t - \delta_{k1}\mu_k(r\cos\alpha_k - z\sin\alpha_k))} + A_l^{(k)} e^{i(\omega t - \delta_{t1}\mu_{t1}(r\sin\alpha_l^{(k)} - z\cos\alpha_l^{(k)}))}, & l, k = 3, \end{cases}$$

$$\psi_1 = A_3^{(k)} e^{i(\omega t - \delta_4(r\sin\alpha_4^{(k)} - z\cos\alpha_4^{(k)}))},\quad \psi_2 = A_4^{(k)} e^{i(\omega t - \delta_4\mu_4(r\sin\alpha_4^{(k)} - z\cos\alpha_4^{(k)}))}. \quad (5)$$

Using eq. (4) in eq. (5), after a long calculation, the solid displacements are obtained as follows

$$u_1 = \begin{cases} -i\{\delta_{k1}\phi_{(k)}\sin\alpha_k + \sum_{t1=1}^{2}\delta_{t1}\phi_t^{(k)}\sin\alpha_{t1}^{(k)} - \delta_4\phi_4^{(k)}\cos\alpha_4^{(k)}\} & k = 1,2, \\ i\{\delta_{k1}\phi_{(k)}\cos\alpha_k + \sum_{t1=1}^{2}\delta_{t1}\phi_{t1}^{(k)}\sin\alpha_{t1}^{(k)} - \delta_4\varphi_4^{(k)}\cos\alpha_4^{(k)}\} & k = 3, \end{cases}$$

$$w_1 = \begin{cases} -i\{\delta_{k1}\phi_{(k)}\cos\alpha_k + \sum_{t1=1}^{2}\delta_{t1}\phi_{t1}^{(k)}\cos\alpha_{t1}^{(k)} + (\phi_{t1}^{(k)}/r) + \delta_4\phi_4^{(k)}\sin\alpha_4^{(k)}\} & k = 1,2, \\ i\{\delta_{k1}\phi_{(k)}\sin\alpha_k + \sum_{t1=1}^{2}\delta_{t1}\phi_{t1}^{(k)}\cos\alpha_{t1}^{(k)} + (\phi_{t1}^{(k)}/r) + \delta_4\varphi_4^{(k)}\sin\alpha_4^{(k)}\} & k = 3. \end{cases} \quad (6)$$

Using these displacement components in eq. (2), the non-zero stress components and fluid pressure are calculated and are given by

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$$(\sigma_{rz})_1 = \left(\begin{array}{l} \{\alpha a_1 a_3 + 2a_1 a_3(\mu_L - \mu_T) + \beta a_1^3 a_3\} \\ \left\{ \begin{array}{l} \{\delta_{k1}^2 \phi_{(k)} \sin^2 \alpha_k - \sum_{l=1}^{2} \delta_l \phi_l^{(k)} \sin^2 \alpha_l^{(k)} - \delta_4^2 \phi_4^{(k)} \cos 2\alpha_4^{(k)}\} \\ \{\delta_{k1}^2 \phi_{(k)} \sin 2\alpha_k - \sum_{l=1}^{2} \delta_{t1} \phi_{t1}^{(k)} \sin 2\alpha_{t1}^{(k)} - \delta_4^2 \phi_4^{(k)} \cos 2\alpha_4^{(k)}\} \end{array} \right\} + \{\mu_T + (\mu_L - \mu_T)(a_1^2 + a_3^2) + \beta a_1^2 a_3^2\} \\ \left\{ \begin{array}{l} \left\{ \begin{array}{l} \{\delta_k^2 \phi_{(k)} \sin 2\alpha_k - \sum_{l=1}^{2} \delta_l^2 \phi_l^{(k)} \sin 2\alpha_l^{(k)} - (\phi_l^{(k)} / r) + \delta_4^2 \phi_4^{(k)} \cos 2\alpha_4^{(k)} + (i/r)\delta_l \phi_l^{(k)} \sin \alpha_l^{(k)}\}, \\ \{\delta_k^2 \phi_{(k)} \cos 2\alpha_k - \sum_{l=1}^{2} \delta_l \phi_l^{(k)} \sin 2\alpha_l^{(k)} - (\phi_l^{(k)} / r^2) + \delta_4^2 \phi_4^{(k)} \cos 2\alpha_4^{(k)} + (i/r)\delta_{t1} \phi_{t1}^{(k)} \sin \alpha_l^{(k)}\} \end{array} \right\} + \\ \left\{ \begin{array}{l} \{\delta_k^2 \phi_{(k)} \sin 2\alpha_k - \sum_{l=1}^{2} \delta_l \phi_l^{(k)} \sin 2\alpha_l^{(k)} - \delta_4^2 \phi_4^{(k)} \cos 2\alpha_3^{(k)}\} \\ \{\delta_k^2 \phi_{(k)} \cos 2\alpha_k - \sum_{l=1}^{2} \delta_l \phi_l^{(k)} \sin 2\alpha_l^{(k)} - \delta_4^2 \phi_4^{(k)} \cos 2\alpha_3^{(k)}\} \end{array} \right\} \end{array} \right\} + \\ \{\alpha a_1 a_3 + 2a_1 a_3(\mu_L - \mu_T) + \beta a_1 a_3\} \left\{ \begin{array}{l} \{\delta_{k1}^2 \phi_{(k)} \sin^2 \alpha_k - \sum_{l=1}^{2} \delta_t \phi_t^{(k)} \sin^2 \alpha_t^{(k)} - \delta_4^2 \phi_4^{(k)} \cos 2\alpha_4^{(k)}\}, \\ \{\delta_{k1}^2 \phi_{(k)} \sin 2\alpha_k - \sum_{l=1}^{2} \delta_{t1} \phi_{t1}^{(k)} \sin 2\alpha_{t1}^{(k)} - \delta_4^2 \phi_4^{(k)} \cos 2\alpha_4^{(k)}\}, \end{array} \right\} \begin{cases} k, l = 1,2. \\ k, l = 3. \end{cases} \end{array} \right)$$

$$(\sigma_{zz})_1 = \left(\begin{array}{l} \{\lambda + \alpha(a_1^2 + a_3^2) + \beta a_1^2 a_3^2\} \left\{ \begin{array}{l} \{\delta_{k1}^2 \phi_{(k)} \sin^2 \alpha_k - \sum_{l=1}^{2} \delta_l \phi_l^{(k)} \sin 2\alpha_l^{(k)} + \delta_4^2 \phi_4^{(k)} \cos 2\alpha_4^{(k)}\}, \\ \{\delta_{k1}^2 \phi_{(k)} \sin 2\alpha_k - l\phi_l^{(k)} \sin 2\alpha_l^{(k)} + \delta_4^2 \phi_4^{(k)} \cos 2\alpha_4^{(k)}\} \end{array} \right\} + \\ \{\alpha a_1 a_3 + 2a_1 a_3(\mu_L - \mu_T) + \beta a_1 a_3^3\} \left\{ \begin{array}{l} \delta_{k1}^2 \phi_{(k)} \sin 2\alpha_k + \sum_{l=1}^{2} \delta_l \phi_{lt}^{(k)} (\sin 2\alpha_{lt}^{(k)} - \cos^2 \alpha_l^{(k)}) - \delta_4^2 \phi_4^{(k)} \sin \alpha_4^{(k)} \\ \delta_{k1}^2 \phi_{(k)} \cos^2 \alpha_k + \sum_{l=1}^{2} \delta_t \phi_t^{(k)} (\sin 2\alpha_{tl}^{(k)} - \cos^2 \alpha_{lt}^{(k)}) + \delta_4^2 \phi_4^{(k)} \sin \alpha_4^{(k)} \end{array} \right\} + \\ (\lambda + 2\mu_T + 2\alpha a_3^2 + 4a_3^2(\mu_L - \mu_T) + \beta a_3^4\} \\ \left\{ \begin{array}{l} -\delta_{k1}^2 \phi_{(k)} \sin 2\alpha_k + \sum_{l=1}^{2} \delta_l \phi_l^{(k)} (\cos^2 \alpha_l^{(k)} + (1/r)\cos \alpha_l^{(k)} + \sin 2\alpha_l^{(k)}) + \delta_4^2 \phi_4^{(k)} \cos \alpha_4^{(k)} \\ \delta_{k1}^2 \phi_{(k)} \cos^2 \alpha_k + \sum_{l=1}^{2} \delta_t \phi_t^{(k)} (\cos^2 \alpha_l^{(k)} + (1/r)\cos \alpha_l^{(k)} + \sin 2\alpha_{lt}^{(k)}) + \delta_4^2 \phi_4^{(k)} \cos \alpha_4^{(k)} \end{array} \right\} \begin{cases} k = 1,2, l = 1,2. \\ k, l = 3. \end{cases} \end{array} \right)$$

$$\mu_{t1} = \frac{(\rho_{11} R - \rho_{12} Q) - (PR - Q^2) V_{t1}^{-2}}{(\rho_{22} Q - \rho_{12} R)}, t1 = 1,2. \qquad (7)$$

3. BOUNDARY CONDITIONS

Continuity of stress components

$(\sigma_{rz})_1 = 0$ at $z = 0,$

$(\sigma_{zz})_1 = 0$ at $z = 0.$ (11)

Continuity of displacement components

$(u)_1 = 0$ at $z = 0,$

$(w)_1 = 0$ at $z = 0.$

Using these boundary conditions, after some mathematical calculations the ratio of reflected wave to the incidented wave is obtained. in this way we get the reflection coefficients of a dilatational waves and a shear wave.

The reflection coefficients are computed by introducing the non-dimensional parameters given below:

$$a_1 = \frac{P}{H}, a_2 = \frac{Q}{H}, a_3 = \frac{R}{H}, a_4 = \frac{N}{H}, d_1 = \frac{\rho_{11}}{\rho}, d_2 = \frac{\rho_{12}}{\rho}, d_3 = \frac{\rho_{22}}{\rho},$$

$$H = P + 2Q + R,\ \rho = \rho_{11} + 2\rho_{12} + \rho_{22}.\ \mu_l = \frac{(a_3d_1 - a_2d_2) - (a_1a_3 - a_2^2)V_k^{-2}}{(a_2d_3 - a_3d_2)},\ \mu_k = \frac{(a_3d_1 - a_2d_2) - (a_1a_3 - a_2^2)V_l^{-2}}{(a_2d_3 - a_3d_2)},\ V_k = \left(\frac{V_0}{V_k}\right)^2,$$

$V_l = \left(\frac{V_0}{V_l}\right)^2$, $k, l, t = 1, 2, 3,$ are the normalized phase velocities of k^{th} incidented and l^{th} reflected waves respectively. In the above $V_0 = \sqrt{\frac{H}{\rho}}$ is reference velocity, V_1 and V_2 are the velocities of dilatational waves of first and second kind respectively, and V_3 is the velocity of shear wave. For illustration purpose, two types of poroelastic solid cylinders are used, namely, cylinder-I made up of sandstone saturated with water [14], cylinder-II made up of sandstone saturated with kerosene [15]. The physical parameter values are given in Table-1. Employing these values in eq. (9), reflection coefficients are computed as a function of angle of incidence and the results are depicted in figures 2 to 10. The notations Rcyl-I, Rcyl-II, used in the figures representing the reflection coefficients of cylinder-I, reflection coefficient of cylinder-II,

Table-1: Material Parameters

Material Parameters	a_1	a_2	a_3	a_4	d_1	d_2	d_3
Cylinder-I	0.843	0.065	0.027	0.234	0.901	−0.001	0.1
Cylinder-II	0.96	0.006	0.029	0.412	0.876	0	0.124

Incident fast dilatational wave

The reflection coefficient $Z_1^{(1)}$ and transmission coefficient $T_1^{(1)}$ of a fast dilatational wave (P-I wave) are calculated. In this the reflection coefficients for cylinder-I and cylinder-II increases gradually as angle of incidence increases.

Incident slow dilatational wave

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The reflection coefficient for cylinder-I are periodic and cylinder-II decreases. The reflection coefficient for cylinder-I decreases and cylinder-II increases.

The reflection coefficients for cylinder -I and cylinder-II increases. in this the reflection coefficient for Cylinder-I decreases, for cylinder-II increases. The transmission coefficient for cylinder-I decreases and for cylinder-II increases.

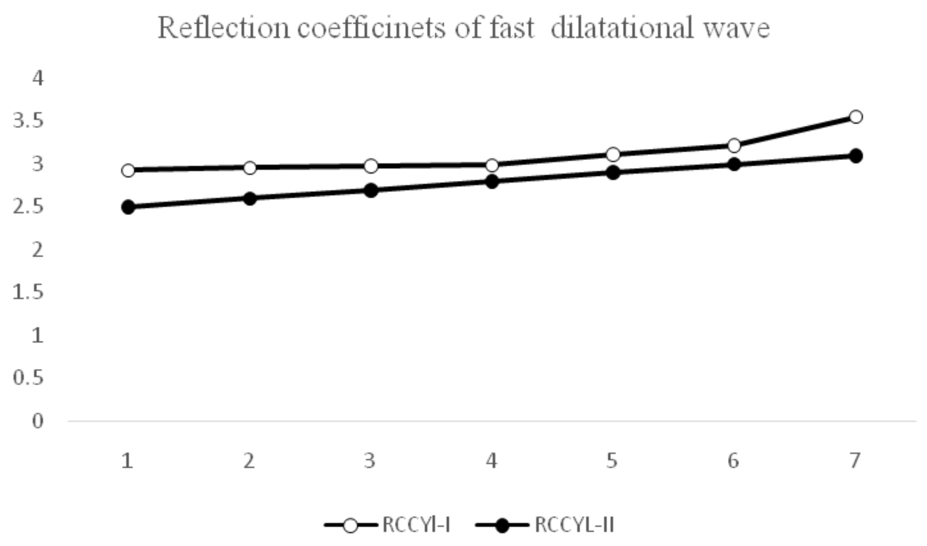

Fig: Reflection coefficients of fast dilatational wave

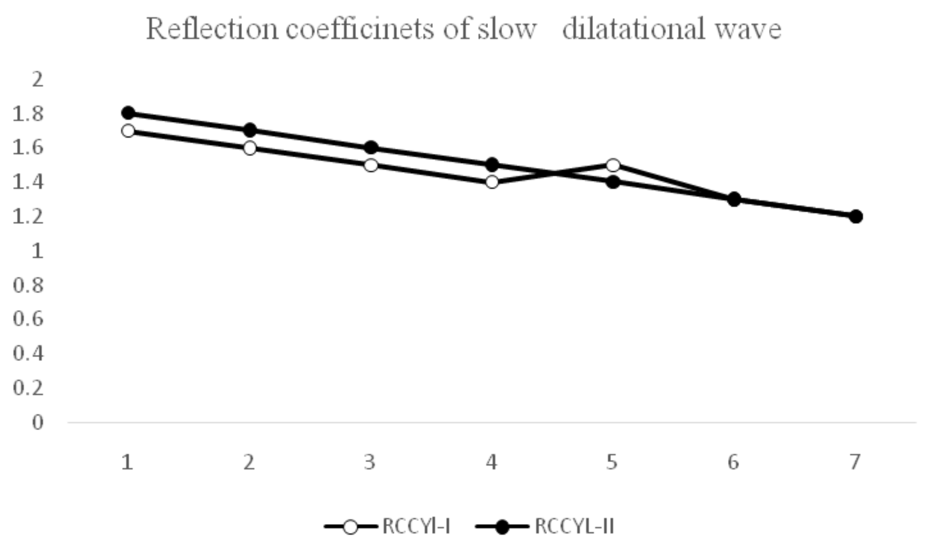

Fig: Reflection coefficients of slow dilatational wave

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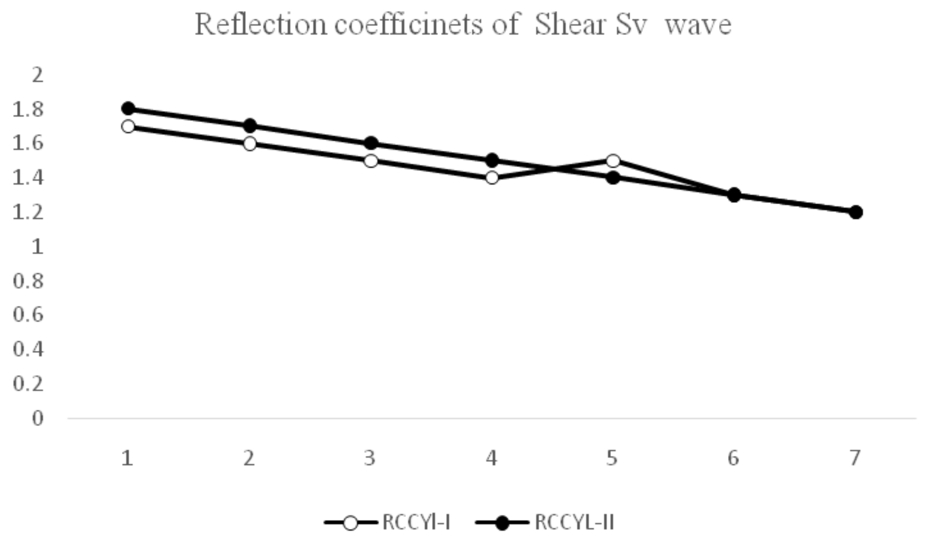

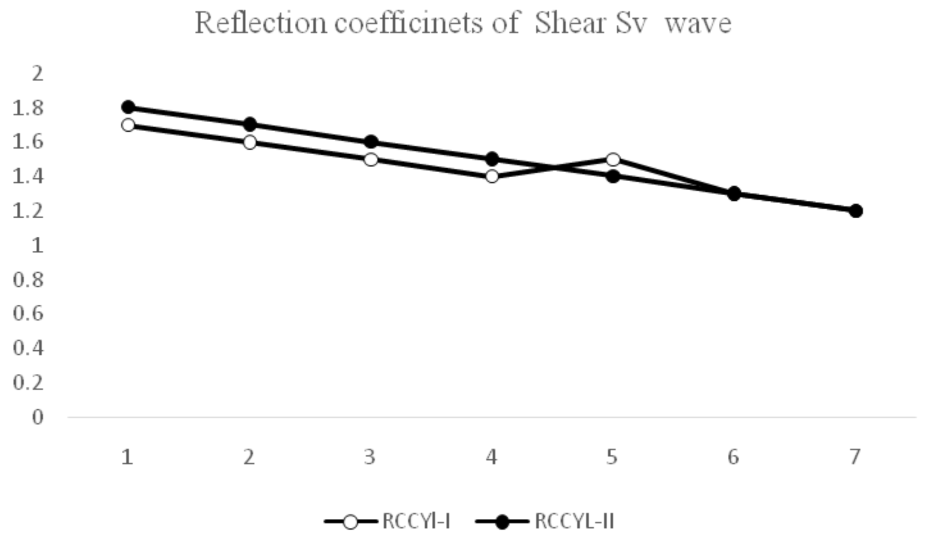

Fig: Reflection coefficients of Shear SV wave

4. CONCLUSION

The reflection coefficients of two dilatational waves and a shear wave are calculated for a two layered problem and the results are depicted in the figures , these represents the reflection coefficients the reflection coefficients in general increases for a dilatational waves and decreases for a shear wave.

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